

THE
DESCRIPTION and USE
OF AN

ORRERY



New CONSTRUCTION,

Representing in the various Parts of its
Machinery all the MOTIONS and PHENO-
MENA of the

PLANETARY SYSTEM;

To which is subjoin'd

A MATHEMATICAL THEORY

For calculating the WHEEL-WORK to the greatest
Degree of Exactness.

BY BENJAMIN MARTIN. *K*

L O N D O N:

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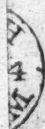
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P R E F A C E.

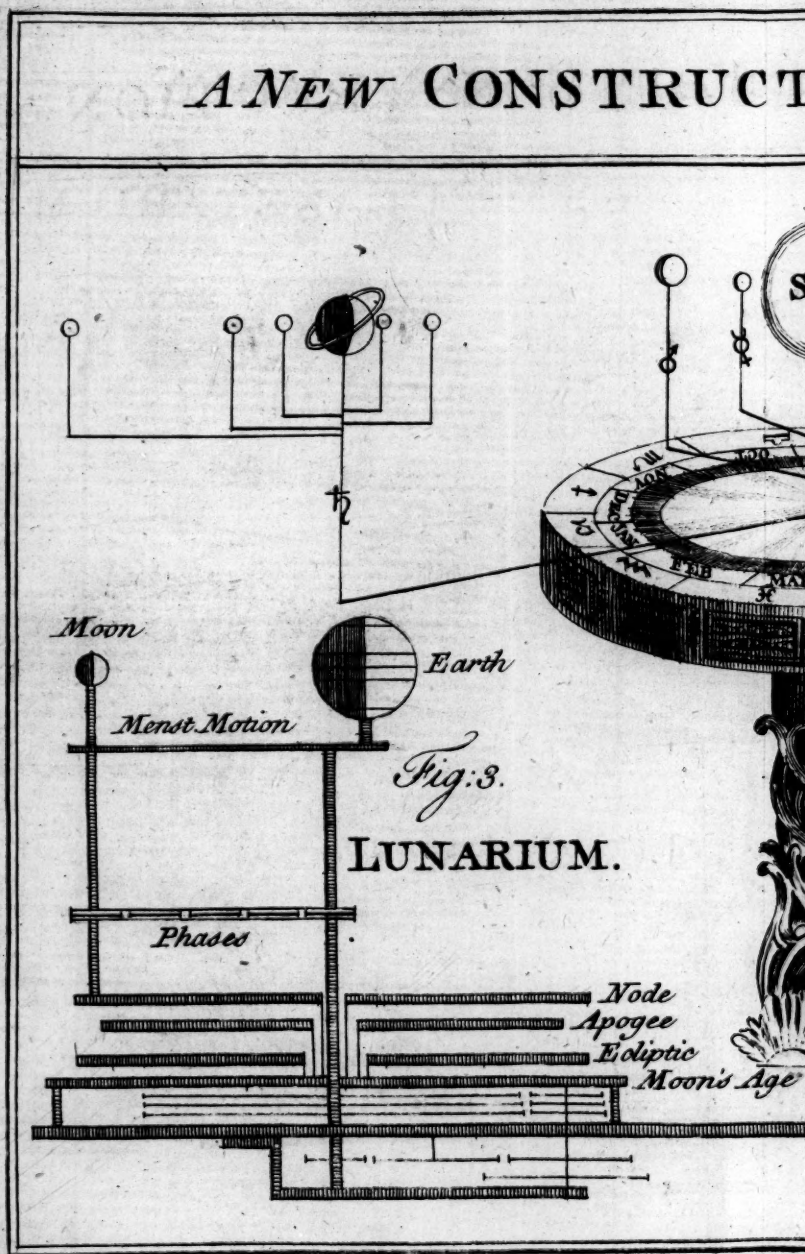
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T H E

WILLIAM CONZTBY



A NEW CONSTRUCT



CONSTRUCTION of an ORRERY.

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PLANETARIUM.

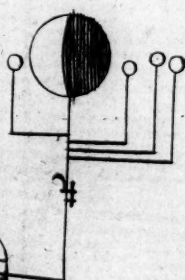
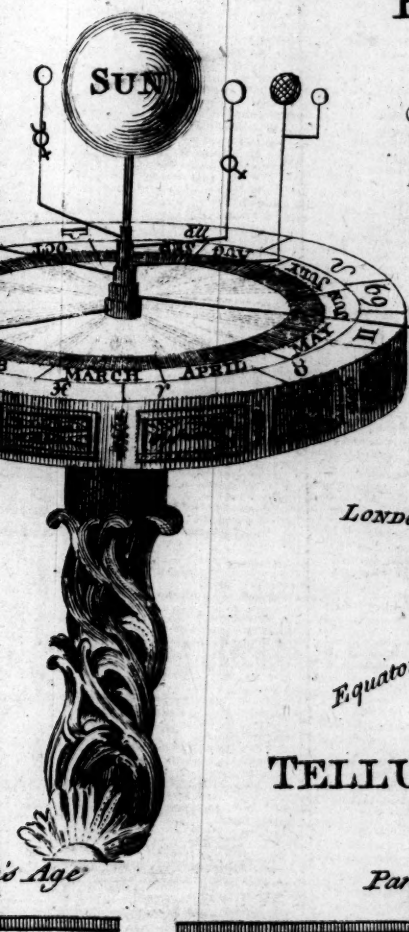
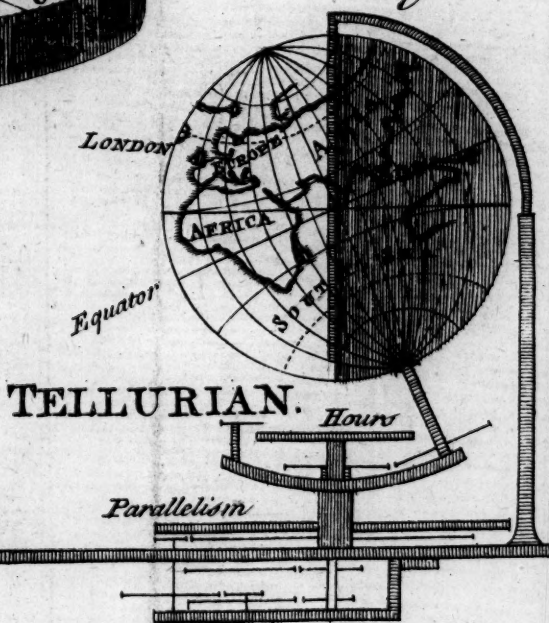
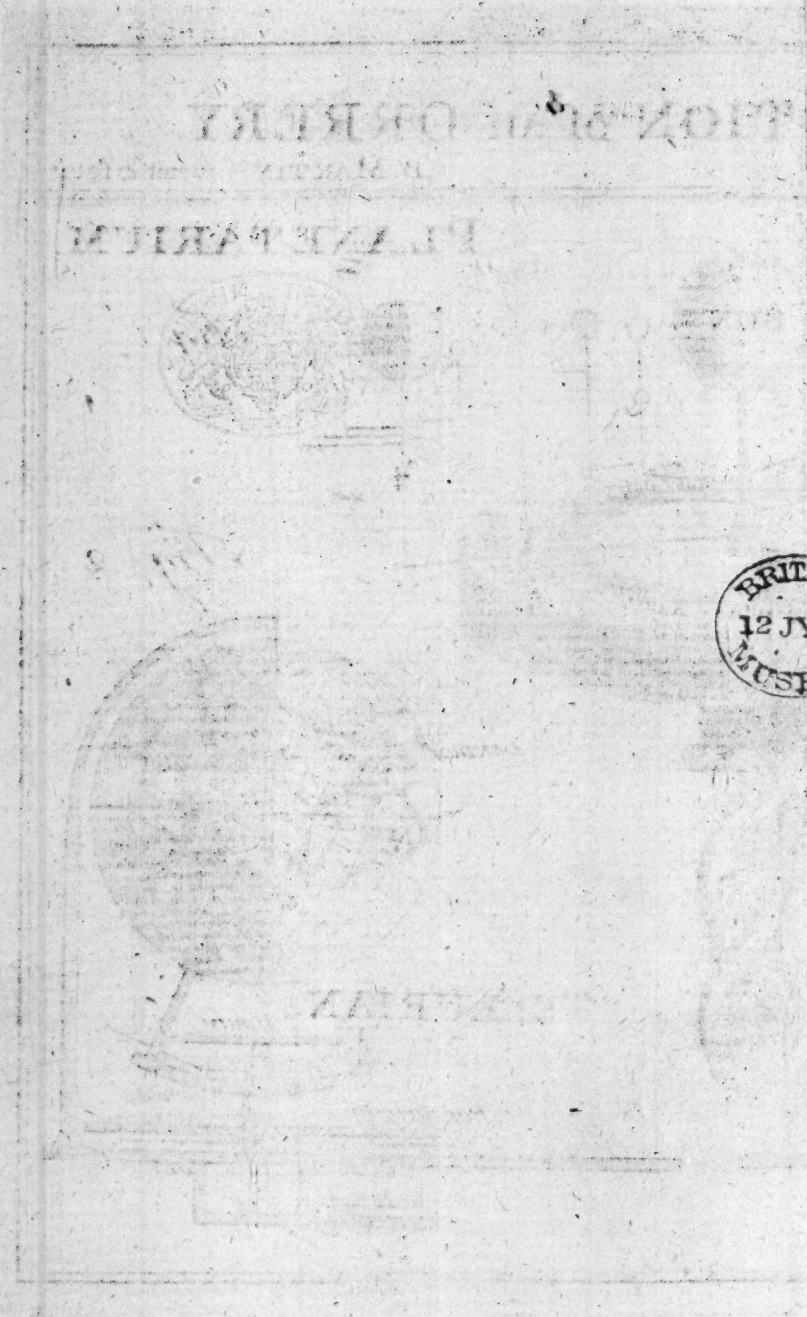


Fig. 1.

Fig. 2.



TELLURIAN.



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T H E



T H E
DESCRIPTION and USE
OF AN
O R R E R Y,
Of a New CONSTRUCTION, derived from the
most exact CALCULATIONS founded on a
MATHEMATICAL THEORY.

N this *New Construction* of an ORRERY, three Things are proposed to recommend it to the Public; (1.) A Greater Number of *Celestial and Terrestrial Phænomena* will be represented in, and by its several Parts, than the Common Construction will admit of; indeed, none of any Moment are here omitted. (2.) The Disposition of the several Parts will allow of a much nearer Congruity with, and a more perfect Resemblance of the numerous Phænomena of the System. (3.) The EXPENCE of an ORRERY in this Construction is far less than that of the Common Sort; *a natural and instructive Simplicity* being here preferred to *useless and costly Magnificence*.

The Component Parts of this ORRERY are as follow:
(1.) A PLANETARIUM, exhibiting the DISPOSITION,
MOTION, and ASPECTS, of all the *Primary PLANETS* of
the SYSTEM, (2.) A TELLURIAN, shewing the *Annual*
B

nual and *Diurnal* MOTIONS of the EARTH, and all the PHÆNOMENA depending thereon, relative to the variable Lengths of DAYS and NIGHTS; and Vicissitudes of SEASONS of the YEAR, in the different CLIMATES of the EARTH, &c. (3.) A LUNARIUM, representing the *Menstrual* MOTION of the EARTH and MOON; the *Periodical* and *Synodical* MONTHS; the *direct* Motion of the APSIDES; the *Retrograde* Motion of the NODES; the PHASES of the MOON; the Manner of *Solar* and *Lunar* ECLIPSES; the *Geocentric* ASPECTS of the PLANETS, &c. (4.) The JOVIAN SYSTEM of four MOONS, and (5.) The SATURNIAN SYSTEM of five MOONS, and RING, all in Motion. (6.) A HELIOTROPE, shewing the MOTION of the SUN about its Axis, by its *Maculæ* or Spots. Besides other MACHINERY shewing the *Stations*, *Retrogradations*, *Transits*, and other ASPECTS of the Planets.

I. Of the PLANETARIUM.

THE PLANETARIUM is the Body or Basis of the whole Machine, or ORRERY. (Fig. 1.) It contains the Wheel-Work by which the six Primary Planets are carried about the Sun in the Center, in their proper or stated PERIODS of Time. These Planets are easily put on, and taken off from their respective Sockets occasionally. They are round Ivory Balls, having one Hemisphere (next the Sun) *white*, the other *black*, to shew their respective Phases to each other. Their Order or Disposition is as follows: In the Center of the System is a large Brass Ball or Globe properly gilt, to represent the SUN, about which moves the Planets MERCURY ☿, VENUS ♀, the EARTH ☉, MARS ♂, JUPITER ♃, and SATURN ♄, each supported on a proper *Brachium*, or Arm.

About the *Primary* PLANETS are placed their *SECONDARIES*, *Satellites*, or MOONS; thus about the Earth is
one;

Of an ORRERY of a New CONSTRUCTION. 3

one; about Jupiter are *four*; and about Saturn, *five*, besides his RING; these are moveable at Pleasure by the Hand in the ordinary Construction of the Planetarium; but in that which is most compleat, they are moved by Wheel-Work in common with their Primaries, and in their true periodical Times.

Upon the PLATE of the *Planetarium* are placed in two apposite Circles, the ECLIPTIC and the KALENDAR of MONTHS; by Means whereof the Planets are easily set to their mean Places in the Ecliptic for any Day of the Year. Then by turning the Winch, or Handle, they all move about the Sun from WEST to EAST with the same respective *Velocities* of Motion, and *periodical* TIMES, as they have in the Heavens. And thus you have a just Representation of the true *Pythagorean*, *Copernican*, or *Solar* SYSTEM.

Then to represent the *Ptolomaic*, or *Vulgar* SYSTEM, you take the Sun from the Center, and in its Stead put on a smaller Ivory Ball with Circles to represent the Central EARTH; and then taking off the Earth from the 3d Arm, you place a small Brass Ball in it's Place, to denote the Sun revolving (with the rest of the Planets) about the Earth at Rest in the Center. By these Constructions of the two Systems, the TRUTH of the one, and the *Absurdity* of the other, will be rendered extremely evident by the *Phænomena* arising from each.

How very near the Mean Motions of the Planets in the ORRERY approach to those in the HEAVENS, will appear by the following Table, wherein the first Column is of the Planets; the 2d shews their Periodical Times about the Sun; and the 3d shews how much each Planet's Motion in the ORRERY differs therefrom in the Space of 20 Years.

A DESCRIPTION and Use

Planets.	Periodical Times.				Diff. in 20 Years.			
	D.	H.	"	"	"	"	"	"
MERCURY	87.	23.	14.	34.	16.	56.	16.	too slow
VENUS	224.	16.	41.	30.	6.	28.	40.	too fast
EARTH	365.	5.	49.	25.	17.	48.	0.	too fast
MARS	686.	22.	18.	19.	7.	53.	24.	too fast
JUPITER	4330.	8.	35.	0.	4.	40.	48.	too slow
SARURN	10750.	13.	14.	42.	1.	37.	12.	too fast

II. Of the TELLURIAN.

The Second Part of this Orrery I call a TELLURIAN, (Fig. 2.) because it shews most accurately and evidently all the *Phænomena* arising from the *Annual* and *Diurnal* MOTIONS of the EARTH, in a *Terrestrial* GLOBE full *Three Inches* in a Diameter; upon which all the Parts of the terraqueous Surface are distinctly delineated, and the EQUATOR, ECLIPTIC, and MERIDIAN, divided into their proper Degrees; the HOUR CIRCLES drawn thro' every 15 Degrees of the Equator; the PARALLELS, also, thro' every 10th Degree of North and South Latitude; with the TROPICS of Cancer, and Capricorn; the *Arctic* and *Antarctic* Circles, &c. as in the larger Globes.

All the *Continents*, *Kingdoms*, and *Islands*, of Note, are easily seen; as also all the *Oceans*, *Seas*, and *Bays*; so that the *Annual Motion* of the Earth about the Sun, and the *Diurnal Motion* upon its Axis will exhibit the same Appearances as our real Earth would do if beheld at a proper Distance from its Surface; for whatever relates to changeable SEASONS, and the variable Lengths of DAYS and NIGHTS in every Part of the Year, and in every particular *Country*, *Climate*, or *Latitude*, will be viewed equally alike in both.

To this End the Axis of the Earth has a perfect *Parallelism* and *Inclination* to the Plane of the *Ecliptic*; there
is

Of an ORRERY of a NEW CONSTRUCTION. 5

is also a *Circle of Illumination* about the Globe, dividing the *enlightened* from the *dark* HEMISPHERE; and by a Dial-Plate of the 24 Hours just under the Globe, it will appear at what Hour the Sun *rises* and *sets* to every Part of the Earth, and on every Day of the Year.

There is likewise a proper Index to shew when it is NOON, or the Sun is upon the Meridian of any particular Place; by the same Index is also shewn what *Sign and Degree* of the Ecliptic the Sun is in for every Day; the Parallel of Declination it describes; and the Length of the *Diurnal* and *Nocturnal* Parts of that Parallel in the light and dark Hemispheres.

For some Purposes it will be requisite to have only the *Diurnal Motion* of the Earth, and then there is a Provision made for *stopping the Annual Motion* at Pleasure, by moving sideways a Brass Studd. Some Phænomena likewise require a *greater Velocity* of the Earth about its Axis, than others; and therefore by moving another Studd you effect that; for thus you will have the Earth move *once round* with every Turn of the Handle, or only $\frac{1}{3}$ Part round, or 3 Hours, by one Turn.

About the Socket for the Earth there is a fix'd Socket upon which a Circle is firmly placed, of Six Inches Diameter, whose Periphery is divided into a proper Number of Teeth, for giving Motion to the Machinery of the TELLURIAN; upon the silver'd Surface of this broad Circle are engraved the ECLIPTIC with its Signs and Degrees, and the KALENDAR of Months and Days adapted thereto.

When the Tellurian is used, it is to be put upon the Socket for the Earth, and placed exactly over the Degree of the Ecliptic which the Earth possesses that Day, or so that the Index at the End of the Arm (opposite to the Earth) may point to the Sun's Place in the Ecliptic; and then

then screwing it fast upon the Socket and turning the Winch, you will see the *Annual and Diurnal Motion* commence, and continue, every way analogous to those in Nature.

III. Of the LUNARIUM.

After the same Manner, you place upon the same Socket, the LUNARIUM; (Fig. 3.) whose Machinery is also put into Motion by the Teeth on the fix'd Circle. The Motions shewn by the *Lunarium* answer all the Phænomena of a Satellite or Moon, revolving about its Primary, while that is carried about the Sun. These Motions and Appearances are as follow, viz.

First; the *Menstrual MOTION* of the EARTH and MOON about the Common CENTER of GRAVITY between them.

Secondly; The *Circular MOTION* of this CENTER of GRAVITY about the Sun, which describes the *True Annual Orbit*; and in which the Earth has a very irregular or rather a *vermicular Motion* from one Side to the other; being in each Month *nearer to*, and *farther from* the Sun.

Thirdly; The *Monthly Motions* of the Moon, viz. the *Periodical Month* in which the Moon describes exactly the Ecliptic; and the *Synodical Month*, which is the Space of Time between two *New Moons* or *Conjunctions*.

Fourthly; The *Annual parallel MOTION* of the ECIP-TIC; shewing the Place of the MOON, the NODE and APOGEE; as also the *Geocentric Motions, Places, and Aspects* of the SUN, and all the PLANETS.

Fifthly; The *retrograde MOTION* of the NODES with the INCINATION of the *Lunar Orbit*, and the DEGREES of her LATITUDE from the Ecliptic in every Part.

Sixthly;

Of an ORRERY of a NEW CONSTRUCTION. 7

Sixthly; The *Direct* MOTION of the APOGEE, or the *Line* of the *Apsides*; with the Situation of the *Elliptical Orbit* of the Moon and Place of the Apogee in the *Ecliptic* at all Times.

By such a Variety of Motions in the LUNARIUM, it will be very easy to see the *Rationale* of most of the *Lunar Irregularities* and variable *Phænomena*; also every thing relative to the Nature and Doctrine of ECLIPSES, both *Solar* and *Lunar*; *Total*, *Annular*, and *Partial*.

Moreover the Mechanism for the PHASES of the Moon is such as always causes her to shew the *same FACE* to the Earth, without *hiding* any Part of her Surface; so that every Phase of the Moon in the Orrery, is every way similar to that in the Heavens at the same Time.

The great Exactness in the Motions of the LUNARIUM will appear by the Table below, exhibiting the Difference in Motion in 20 Years between the same in the Heavens and in the Orrery.

	D.	H.	
MOON'S Synod. Revol.	29.	12. 44'. 3"	28'. 3". too fast
An. Mot. of APOGEE	40°.	39. 50'. 00"	3'. 27". too fast
Ditto of the NODE	19°.	19. 43'. 00"	4'. 6". too slow

IV. Of the JOVIAN SYSTEM.

By this Part the Motions of Jupiter's 4 Moons or Satellites, are shewn as nearly corresponding to their Motions in the Heavens, as the latest Observations upon them will admit of.

This Piece of Machinery is put into Motion by the Teeth in the Periphery of a large Circle placed firmly on a fix'd Socket, enclosing the Socket of *Jupiter*, and upon this the *Jovian System* is to be placed in such a Manner, that Jupiter's Place in the *Ecliptic* of the Orrery may be the same with his *Heliocentric* Place in the Heavens at
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the given Time. And then, by placing the Studd for the flow Motion, and turning the Handle, you will see JUPITER move in his proper Orbit, and his SATELLITES at the same Time describing their respective Orbits about him, as the Center of their Motions, and in the PERIODS of TIME as specified in the Table below.

Satel.	Periodical Times.				Distances.
	D.	H.	'	"	
1	1.	18.	27.	34.	5,67
2	3.	13.	13.	42.	9,00
3	7.	3.	42.	36.	14,38
4	16.	16.	32.	09.	25,3

The Distances of the Satellites have the same Proportion here as in the Heavens; which are expressed (as in the Table) in Semidiameters and Decimal Parts of Jupiter's Globe; therefore since this *Jovian Planetarium* bears so great a Resemblance to that of the *Solar One*, (in Fig. 1.) I think a Figure of it will be needless.

Lastly; The ROTATION of JUPITER about his Axis is here shewn to be the same as in the Heavens, viz. in 9 Hours, 55 Minutes, and 54 Seconds; by which all the Phænomena of his DAYS, and NIGHTS, SEASONS, &c. are exhibited.

V. Of the SATURNIAN SYSTEM.

The *Brachium* or Arm which carries the *Saturnian SYSTEM*, is skrew'd fast upon the Socket for SATURN, and has it's Machinery put into Motion by the Teeth upon the Edge of the Plate of the Planetarium; the Primary being first carefully set to its true Place in the Ecliptic for the given Time.

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Of an ORRERY of a NEW CONSTRUCTION. 9

By turning the Handle, the Primary and its Satellites all move similarly to their Originals in the Heavens; about an Axis *inclined to the Plane of the Ecliptic* in an Angle of 31 Degrees, as that of Saturn and his System is found to be by the apparent elliptic Motions of the Satellites, and the Phases of the Ring.

The *Periodical* MOTIONS of the five Moons are respectively as in the Table below; and also their *Distances* from Saturn's Center (measured in Semidiameters of the *Ring*) in all but one, viz. in the 5th, for otherwise the Diameter of Saturn and his Ring, and indeed of the whole System, must be very small.

During the whole Revolution of SATURN, his Ring, by its parallel Motion, exhibits the very same Phases as are seen in the Heavens at the same Times. As no *Diurnal* MOTION of SATURN about its Axis has been yet discovered, so none is here represented.

Satel.	Periodical Times.				Distances.
	D	H.	"	"	
1	1.	21.	18.	27.	2,097
2	2.	17.	41.	22.	2,686
3	4.	12.	25.	12.	3,752
4	15.	22.	41.	12.	8,698
5	79.	7.	47.	0.	25,348

VI. Of the HELIOTROPE.

The HELIOTROPE is a small SYSTEM of MECHANISM fixed upon the Axis of the *Planetarium*, just under the Sun, and by which the Motion of the SUN about its Axis is shewn in 25 Days, 15 Hours, 16 Minutes, as it is observed in the Heavens, by the regular Motion of the *Maculae* or Spots on his Disk there, and as they here appear in the Orrery.

The Motion in the HELIOTROPE originates from the Motion of the Planet MERCURY, as being connected

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therewith. This Part may be easily put on, or taken off, as Occasion requires.

The various Pieces of Machinery hitherto described, though they make the greatest and most essential Part of the ORRERY, yet are not sufficient to represent all the Phænomena of the Heavens; some of which require additional Contrivances, Circles, and particular Machinery; the chief of which here follow:

First; The INCLINATION of the Planes of the Orbits of the Planets to the Plane of the Ecliptic, is an Affair of too great Nicety and Minuteness to be expressed by Machinery, as will be evident in the Table following of the said INCLINATION, Place of the *ascending* NODE, and the *Aphelium* peculiar to each Plane, according to Dr. HALLEY for the Year 1800.

PLANETS	INCLINATION	Ascend NODE	APHELIUM
MERCURY	6°. 59. 20"	8. 16°. 10. 40"	7. 14°. 11. 02"
VENUS	3. 23. 20.	II. 14. 49. 33	6. 59. 44
MARS	1. 51. 00.	8. 18. 28. 02	2. 30. 00
JUPITER	1. 19. 10.	6. 8. 57. 30	11. 33. 48
SATURN	2. 30. 10.	6. 21. 35. 00	7. 29. 13. 20

Such small *Inclinations* as those for *Mars*, *Jupiter*, and *Saturn* could not be sensibly shewn by Machinery, tho' those of *Venus* and *Mercury's* Orbits might; but the Expence would be not a little; and the End may be answered, I think, full as well by CIRCLES distinctly drawn for each Planet upon the Plate of the *Planetarium*, wherein the Position of the NODES and *Aphelia* might be mark'd; and the LATITUDE of the Planet expressed for every Part of its Orbit, *North* or *South*.

Secondly; The PROPORTION of the Planet's DISTANCES from the SUN'S CENTER may be shewn in the 4th list, viz. *Mercury*, *Venus*, the *Earth*, and *Mars*, as also their PROPORTION of MAGNITUDE to each other, by a parti-

Of an ORRERY of a New CONSTRUCTION. IT particular Set of *Brachia* and Balls for that Purpose; but it would be to little Purpose to attempt any thing of this sort for the Whole System.

Thirdly; The *Direct* and *Retrograde* MOTIONS and STATIONS of the Planets may be much more naturally represented than usually in *Orreries*, by a particular Contrivance of a *Radius Vector*, shewing the *True Motion* of the Planet about the Sun, and its apparent Motion among the fix'd Stars at the same Time.

Fourthly; An *Apparatus* is easily adapted to the *Planetarium*, to shew all the *Phænomena* of the TRANSITS of MERCURY and VENUS over the FACE of the SUN, such as I shewed in Public to *Thousands* on the late memorable Instance of 1769.

Fifthly; The Nature and Manner of the PRECESSION of the EQUINOXES, and thereby the Differences between the *Sidereal* and *Tropical* YEAR, as also the *Apparent direct* MOTION of the FIX'D STARS may be rendered very easy to conceive, by what may be properly called A *Platonic GLOBE*, whose Axis, Equinoxes, Co-lures, &c. retrograde, like those of the Earth, thro' the Great *Platonic* YEAR.

I have not included the *Phænomena* of COMETS in the foregoing Idea of an ORRERY; the great *Eccentricity* of their Orbits will by no means admit them into such a Construction. They must therefore be shewn in a particular Machine for that Purpose, and which is properly called a COMETARIUM.

Neither have I mentioned a SPHERE or *Hemisphere* circumscribing the ORRERY, because there is really no *such Thing in Nature*. The ORRERY I propose is a bare Representation of the *Solar* SYSTEM in its native Simplicity, and is, in its self, sufficiently grand, and pompous; it stands in Need of none of the useless, ex-

penfive, and cumbersome Embellishments of Art. The DOCTRINE of the SPHERE has little or no Connection with the *Construction* and *Use* of an ORRERY. The SPHERE is only a *Compages* of imaginary Circles; the ORRERY consists of nothing but Realities, which compose the Planetary Part of the *Mundane SYSTEM*. The Use of the CIRCLES of the SPHERE is best taught upon the Globes, as the Reader will see in my TREATISE on that Subject.

However, Gentlemen may have the ORRERY constructed in what Manner soever they chuse, with or without a Sphere, or Hemisphere; with the *Saturnian* or *Jovian System of Machinery*, or without it; with a full *Lunarium*, or otherwise as they please; and the Prices will be proportioned to the Work.

The CALCULATION of the WHEEL-WORK of every Part is founded on the following *Mathematical THEORY*, which I was induced to subjoin, lest any one should think I desire my *Word only* may be taken for the Truth of what I have asserted. Here the Reader will find a more easy and (I hope) perspicuous *Demonstration* of the Method of constructing Wheel-Work for an ORRERY, than that of the celebrated HUGENIUS in his *Automaton*, who is the only Author I have seen upon this Subject; but he has not a Word about the *Motions* of the *Lunar Node* and *Apogee*; nor about the *Movements* of *Jupiter's* or *Saturn's Moons*. I believe there is not one superfluous Wheel in the whole Orrery, perfect as here proposed. Nor can I think of any wanting; for it seems very certain there is *no Satellite about Venus*; and as to her *Rotation* about her Axis, it is so very uncertain a Matter, that Machinery to represent such a Phænomenon would be very nugatory, if not ridiculous.

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T H E
THEORY of CALCULATIONS
FOR THE
WHEEL-WORK of an ORRERY
TO THE
Most extreme Degree of *Accuracy*.

1. **T**HE THEORY of an ORRERY depends almost wholly upon the Doctrine of *Commensurable Numbers* and *Fractions*, as will appear in the Sequel of this Discourse. To understand which, the following *Definitions* and *Axioms* will be necessary.

2. Definition I. One Number, or Quantity, is said to measure another, when it is exactly contained in that other, without any Remainder; as 3 is contained in 12 just 4 times.

3. Def. II. A Number that exactly measures two others, is said to be a *common Measure* to them both; thus 3 is the common Measure of 12, and 21, as it exactly measures both.

4. Def. III. Numbers or Quantities are said to be *commensurable*, or *incommensurable*, as they will, or will not, admit of a *common Measure*.

5. Def. IV. Whole Numbers are said to be *prime* to each other, which admit of no common Measure, but *Unity*; thus 3, 7, 11, &c. are *prime Numbers*.

6. Def.

6. Def. V. When a less Number measures a greater, the less is called an *Aliquot Part* (or *Submultiple*) of the greater; and the greater is said to be a *Multiple* of the lesser.

7. To these Definitions must be subjoined the following AXIOMS, viz.

Axiom I. *If one Quantity measures another, and that other measures a third, then will the first measure the third*: Thus if 3 measures 12, and 12 measures 36, then will 3 measure 36.

8. Ax. II. *If a Number be a common Measure to two others, it will measure both their Sum and their Difference*. Thus because 3 measures 12 and 21, it will measure their Sum 33, and their Difference 9; for since 3 is contain'd 4 times in 12, and 7 times in 21, it must be 11 times contained in their Sum, because $4 + 7 = 11$; and 3 times contained in their Difference, because $7 - 4 = 3$.

9. Ax. III. *If one Number be divided by another, and there be a Remainder, take away the Remainder from the Dividend, and the Divisor will measure the rest*. Thus if 14 be divided by 3, there will remain 2, which taken from 14, there will remain 12, which is measured by 3.

10. By these Definitions and Axioms, we are prepared to give a Demonstration to the following THEOREMS, viz.

THEOREM I.

Let A and B be two Numbers, and A the greater; let A be divided by B, and (without regarding the Quotient) let C be the Remainder; then divide B by C, and let the Remainder be D; again, divide C by D, and let there remain E; and lastly, divide D by E, and let nothing remain; then will this last Divisor E be a common Measure to A and B.

II. For

11. For E measures D, by Hypothesis; and D measures C—E (9.); therefore also E measures C—E (7.); but E measures it's self; therefore E measures both E and C—E; but the Sum of E and C—E is C; therefore E measures C. (8.)

12. Because E measures C, and C measures B—D, therefore E measures B—D; but E measures D, by Supposition; therefore E measures both D and B—D; and therefore it measures their Sum, which is B, the *lesser* given Number,

13. Since E measures B, and B measures A—C, therefore E measures A—C; but E measures C (11.); therefore E measures the Sum of C and A—C, which is A, the *greater* of the given Numbers. Thus then it is proved, that the last Divisor E, without a Remainder, measures both the given Numbers A and B.

THEOREM II.

14. This DIVISOR E is the *greatest common Measure* of A and B. To prove this, let F, if possible, be a greater common Measure to A and B than E is. Then because F measures B, and B measures A—C, therefore F measures A—C; but F measures A by the Supposition, therefore F measures A and A—C; and therefore it measures their Difference C (8.)

15. Since F measures C, and C measures B—D; therefore F measures B—D; but F measures B, by Supposition, therefore F measures B and B—D; and therefore it measures their Difference D.

16. Lastly; Because F measures D, and D measures C—E, therefore F measures C—E; but F measures C (by 14) therefore F will measure both C and C—E; and consequently F will measure E, which is their Difference.

That

That is, a greater Quantity F measures a lesser one E , which is absurd. Therefore E is the greatest common Measure to A and B .

17. Hence if any Number F measures any two Numbers A and B , it will also measure their greatest common Measure E .

18. Since aA and bA are Multiples of A , it is plain their Sum $aA + bA$, and also their Difference $aA - bA$ will be Multiples of the same Quantity A ; for $aA + bA = (a + b) \times A$, and $aA - bA = (a - b) \times A$.

THEOREM III.

19. Suppose A and B any two Quantities whose least common Multiple is C ; then will C measure any other common Multiple D of the same Quantities. If this be denied, it must follow, that after C has measured D as often as is possible, there will still remain a Quantity E less than C : Be it so, and then C will measure $D - E$. Since then A and B both measure C , and C measures $D - E$, A and B will both measure $D - E$; therefore $D - E$ is a common Multiple of A and B , but D is a common Multiple of the same by Supposition; therefore the Difference of D and $D - E$, viz. E , is a common Multiple of A and B (18), but E is less than C , by Supposition; therefore if C does not measure D without a Remainder, it will be possible for A and B to have a common Multiple less than their least, which is absurd. Therefore, &c.

THEOREM IV.

20. If any two unequal Quantities A and B are multiplied together, the Product AB is a common Multiple of both. This may be measured by their least common Multiple

Multiple m , (19.) and let $\frac{AB}{m} = C$; then $AB = mC$,
and $\frac{AB}{C} = m$ the least common Multiple of A and B .

21. Hence because $\frac{AB}{C} = A \times \frac{B}{C} = B \times \frac{A}{C}$, it is evident that $\frac{B}{C}$ and $\frac{A}{C}$ are whole Numbers, or that C is a common Measure to both A and B ; for they are Aliquot Parts of AB , as appears from the Nature and Definition of Multiples.

THEOREM V.

22. But further it appears, that C is the greatest common Measure of A and B ; for if you suppose a greater common Measure D ; then, since $\frac{A}{D}$ and $\frac{B}{D}$ would be whole Numbers, $\frac{AB}{D}$ would be a common Multiple less than their least $\frac{AB}{C}$, which is absurd.

23. Hence the Rule for finding the least common Multiple of any two Quantities or Numbers is, *To multiply them together, and divide the Product by their greatest common Measure, then the Quotient is the least common Multiple sought.* For Instance, suppose the Numbers 12 and 15 were proposed, their greatest common Measure is 3, and their Product 180; then $\frac{180}{3} = 60$, their least common Multiple.

THEOREM VI.

24. Since $\frac{AB}{C} = m$, if we suppose $\frac{mZ}{D} = n$: then will n be the least common Multiple of the two Numbers, m , Z ,
D whose

whose greatest common Measure is D ; and if for m we substitute its value, we have $\frac{ABZ}{CD} = n$, that is, n will be the least common Multiple of the three Numbers A , B , and Z ; for Example, let $A=8$, $B=10$, $Z=12$; then is $C=2$, and $D=4$; and $\frac{ABZ}{CD} = \frac{8 \times 10 \times 12}{2 \times 4} = 120$, the least common Multiple of A , B , and Z , as required. And thus you proceed for four or more Numbers.

25. Hence, if the given numbers A , B , and Z , are prime to each other, then because $C=D=1$, their least common Multiple will be the Product of their Multiplication, viz. $ABZ=n$.

THEOREM VII.

26. When a Fraction $\frac{A}{B}$ is reduced to its least Terms $\frac{D}{E}$, then, because they are still equal, we have $AE=BD$ = least common Multiple of A and B ; because in this Case, if C be the greatest common Measure of A and B , we shall have $\frac{A}{C}=D$, and $\frac{B}{C}=E$, and therefore $AE=BD=\frac{AB}{C}$, the least common Multiple of A and B . (20.)

THEOREM VIII.

27. Let A and B be any two Quantities whose greatest common Measure is C , then if any two unequal Multiples of A and B , as $m A$ and $n B$ be taken and compared, their Difference $m A - n B$ can never be less than C . For since C measures A and B , it measures all their Multiples, for C measures A , and A measures $m A$; therefore C measures $m A$; and therefore also $n B$ (7). But if C measures $m A$ and $n B$, it will measure their Difference $m A - n B$,
(8.)

(8.) therefore no Difference of those Multiples can be less than C.

28. The foregoing Considerations on the *Measures* and *Multiples* of Numbers lay the Foundation of the following Process for the Management and gradual Reduction of large *Fractions* to their *lowest Terms* or Denomination, which is a primary and most essential Article in the THEORY of ORRERY-WORK. This will consist in the Solution of the subsequent

GENERAL PROBLEM.

29. Let A and B be any two given Numbers, of which A is the greater, it is required to find two unequal Multiples m A and n B, whose Difference shall be the least possible.

In this Case, $m A - n B = C$, the greatest common Measure of A and B (by 27.); and the Method of finding this greatest common Measure has already been shewn (10.) and will be illustrated by the following Synopsis, where $A=270$, and $B=112$; then we have, by *Division*,

$$\begin{array}{rcl}
 B=112) 270=A & (2=p & \\
 \underline{224} & & \\
 C=46) 112=B & (2=q & \\
 \underline{92} & & \\
 D=20) 46=C & (2=r & \\
 \underline{40} & & \\
 E=6) 20=D & (3=s & \\
 \underline{18} & & \\
 F=2) 6=E & (3=t & \\
 \underline{6} & & \\
 0. & &
 \end{array}$$

30. By this Operation we find $F=2$ the greatest common Measure of A and B, or of 270 and 112. But we must now find such Multiples of A and B, whose Difference will be equal to F; in order to this a Set of *Artificial Equations* must be formed of the Differences of various

D 2

Multiples

Multiples of A and B, till we arrive at that which shall give the Ratio of A to B in its lowest Terms, or become equal to their greatest common Measure F.

31. To this End it may be considered that any particular Equation as $A=270$, may be formed into a compound differential one, by assuming any other Quantity B, and annexing it to A with a *negative Sign*, and ϕ for it's Co-efficient; thus, $1A-\phi B=270$, the same as before; accordingly, we may put $\phi A-1B=-112$. And then we shall have two Equations to be placed under each other as below:

$$\text{Equat. 1. } 1A-\phi B=270=A.$$

$$\text{Equat. 2. } \phi A-1B=-112=-B.$$

32. We next consider, that when A is divided by B as above, the Quotient is $2=p$, and the Remainder is $46=C$; then C may also be expressed Equation-wise, consisting of the Difference of two Multiples of A and B; for it is evident, that $1A-pB=C$; and it is as plain that this Equation is derived from the other two; for $1A=1A-\phi B$, and $-pB=p \times -B = p \times \overline{\phi A-1B}$; consequently, $\overline{\phi A-1B} \times p + \overline{1A-\phi B} = C = 1A-2B$; we have therefore now three Equations in the following Forms:

$$\text{Equat. 1. } 1A-\phi B=+270=+A.$$

$$\text{Equat. 2. } \phi A-1B=-112=-B.$$

$$\text{Equat. 3. } 1A-2B=+46=+C.$$

33. Again, we see in the Operation (29.) that making B the Dividend, and C the Divisor, we have a Quotient q , and a Remainder D; and that therefore $B-qC=D$; whence $\frac{B-D}{q}=C=A-pB$ (32.) therefore $B-D=qA-pqB$; therefore transposing B, we have $qA-pqB-B=-D=A-pB \times q + \overline{\phi A-1B}=2A-5B$; which is therefore a *fourth Equation* in the Series of Multiples.

Equat.

Equat. 1. $1A - 0B = +270 = +A.$

Equat. 2. $0A - 1B = -112 = -B.$

Equat. 3. $A - 2B = +46 = +C.$

Equat. 4. $2A - 5B = -20 = -D.$

34. From the two last Articles, the Rule for forming each succeeding Equation is evident, viz. *multiply the last Equation by the Quotient of the preceeding Division, to this Product add the preceeding Equation, and the Sum will be the new or subsequent Equation required.* Thus to get a 5th Equation, I multiply the 4th $2A - 5B$ by Quotient $2=r$, and the Product is $4A - 10B$; to this I add the preceeding or 3d Equation $A - 2B$, and the Sum is $5A - 12B = +6 = +E$, which is the 5th Equation required.

35. The sixth Equation is obtained in the same Manner, viz. Multiply the 5th, viz. $5A - 12B$ by the Quotient $3=s$, the Product is $15A - 36B$, to which add the 4th Equation $2A - 5B$, and the Sum is $17A - 41B = -2 = -F$ the Equation required.

36. Then this Equation multiplied by the next Quotient $3=t$, is $51A - 123B$, to which add the 5th Equation, and the Sum will be $56A - 135B = 0$. For at this Step the Division ends, there being no Remainder. Hence the seven Equations will stand as follows:

Equat. 1. $1A - 0B = +270 = +A.$

2. $0A - 1B = -112 = -B.$

3. $1A - 2B = +46 = +C.$

4. $2A - 5B = -20 = -D.$

5. $5A - 12B = +6 = +E.$

6. $17A - 41B = -2 = -F.$

7. $56A - 135B = 0$

37. From hence it appears, that $56A = 135B = 15120$, is the least common Multiple of the Numbers 270 and 112. And therefore the Fraction $\frac{270}{112}$ is, in it's least Terms, $\frac{135}{56}$.

THEO-

THEOREM IX.

38. *The unequal Multiples of A and B which make these Equations, have their absolute Numbers approaching nearer and nearer to the greatest common Measure of A and B; and in the last but one will always be equal to it. Thus the 6th Equation here is $17A - 41B = -2$, or $41B - 17A = 2$, the greatest common Measure of 270, and 112.*

39. *From the Nature and Genesis of these Equations it also appears, that the absolute Numbers are alternately affirmative and negative; and that the respective Co-efficients of the Multiples A and B constitute a Series of Ratio's converging*

towards the Ratio $\frac{A}{B}$; thus the Ratio's $\frac{0}{1}$, $\frac{1}{0}$, $\frac{2}{1}$, $\frac{5}{2}$,

$\frac{12}{5}$, $\frac{41}{17}$, $\frac{135}{56}$; are alternately smaller and greater than

the ratio of A to B: thus $\frac{2}{1}$ is less than $\frac{A}{B}$, and $\frac{5}{2}$ is

greater, but yet nearer; and $\frac{12}{5}$ is less, while $\frac{41}{17}$ is greater,

and very near the ratio of A to B; but the last is exactly

equal to it, viz. $\frac{135}{56} = \frac{A}{B} = \frac{270}{112}$; or $270 : 112 :: 135 : 56$.

THEOREM X.

40. *If the Co-efficients of A and B in any two Equations next to each other be multiplied cross-wise, the Difference of the two Products thence arising will always be equal to Unity. Thus, for Instance, in the 3d Equation we had $1A - pB = C$, and in the 4th, $qA - pq + 1 \times B$ (33.) then $1 \times pq + 1 = pq + 1$, and $p \times q = pq$, which is less than $pq + 1$ by Unity.*

THEO-

THEOREM XI.

41. The Fractions of the Series (39.) are all in their least Terms, for if they were not, the Numbers which compose them must admit of some common Measure C greater than Unity; this Series may be represented in Symbols thus, $\frac{o}{i}, \frac{1}{o}, \frac{p}{p}, \frac{q}{q}, \frac{r}{r}, \frac{s}{s}, \frac{t}{t}$; then since C measures R and r, it will measure Rq and rQ, and their Difference Rq - rQ (27.) But this Difference is equal to unity (40.) therefore C, a Number greater than Unity, measures Unity, which is *absurd*. Therefore the Fraction $\frac{R}{r}$ is in it's lowest Terms. And the same is shewn of all the rest.

THEOREM XII.

42. In the Process of (29.) if instead of the true Quotient 3 at last, the nearest Quotient too little, viz. 2, had been taken, then the 5th Equation added to twice the Sixth, would have produced a Seventh Equation, viz. $39A - 94B = +2$. Where the Multiple of A is now greater than that of B, and $\frac{T}{i} = \frac{94}{39}$.

THEOREM XIII.

43. Again; if to the 6th Equation $17A - 41B = -2$
We add the 7th $- \quad - \quad - \quad 39A - 94B = +2$

The Sum will be the 8th Equat. $56A - 135B = 0$

The same as before in (36.) Therefore the Series com-

pleat will be $\frac{o}{i}, \frac{1}{o}, \frac{p}{p}, \frac{q}{q}, \frac{r}{r}, \frac{s}{s}, \frac{t}{t}, \frac{v}{v}$,

or

or in Numbers $\frac{2}{1}$, $\frac{5}{2}$, $\frac{12}{5}$, $\frac{41}{17}$, $\frac{94}{39}$, $\frac{135}{56}$. Hence

it is evident, that the Ratio $\frac{V}{v} = \frac{A}{B}$ lies between $\frac{S}{s}$ and

$\frac{T}{t}$, but is nearest to the latter; this will be best illustrated

in Numbers, thus; $112 : 270 :: 1 : 2,4107$; and $17 :$

$:: 1 : 2,4118$, and $39 : 94 :: 1 : 2,41025$, therefore

$$\text{We have } \left\{ \begin{array}{l} \frac{S}{s} = \frac{41}{17} = 2,4118 \\ \frac{A}{B} = \frac{270}{112} = 2,4107 \\ \frac{T}{t} = \frac{94}{39} = 2,41025 \end{array} \right\} \begin{array}{l} \text{Difference } 0,0011 \\ \text{Difference } 0,00045 \end{array}$$

So that if the Ratio or Distance between $\frac{S}{s}$ and $\frac{T}{t}$ be di-

vided into 155 equal Parts, the Ratio $\frac{A}{B}$ will be 110

from $\frac{S}{s}$, and only 45 from $\frac{T}{t}$.

THEOREM XIV.

44. In the last Place, it may be proved that no Fraction $\frac{C}{c}$ of a less Denomination than $\frac{T}{t}$ can approach so near to the

Ratio $\frac{A}{B}$. For since $\frac{A}{B}$ lies between $\frac{S}{s}$ and $\frac{T}{t}$, and $\frac{C}{c}$

is supposed to be nearer to $\frac{A}{B}$ than the Fraction $\frac{T}{t}$ therefore

it also lies between $\frac{S}{s}$ and $\frac{T}{t}$. But the Distance or Dif-

ference between $\frac{S}{s}$ and $\frac{T}{t}$ is $\frac{St - Ts}{ts} = \frac{1}{st}$ (40.) and

the

the Difference between $\frac{S}{s}$ and $\frac{C}{c}$ is $\frac{Sc-Cs}{sc}$, which can never be less than $\frac{I}{sc}$; but by the Hypothesis c is less than t , therefore $\frac{I}{sc}$ is greater than $\frac{I}{st}$; therefore the Difference between $\frac{S}{s}$ and $\frac{C}{c}$ will be greater than that between $\frac{S}{s}$ and $\frac{T}{t}$; and therefore the Fraction $\frac{C}{c}$ cannot lie between them, and consequently cannot approach so near to $\frac{A}{B}$ as does the Fraction $\frac{T}{t}$.

THEOREM XV.

45. The foregoing Series of Fractions (39.) may be inverted, and they will stand thus: $\frac{I}{O}, \frac{O}{I}, \frac{I}{2}, \frac{2}{5}, \frac{5}{12}, \frac{17}{41}, \frac{39}{94}, \frac{56}{135}$; but as here $\frac{56}{135} = \frac{B}{A}$, so these Fractions all converge towards the Ratio of B to A; and have all the same Properties with regard to $\frac{B}{A}$, as the other Series has for $\frac{A}{B}$.

46. It may be here proper to illustrate, by a few Examples, the Use of the foregoing THEORY, in reducing very large Fractions to others of a less Denomination, but of nearly the same Value.—Thus the Diameter of a Circle is to its Periphery as 100000000000 to 314159265359, which Ratio let be expressed by B to A, or let $\frac{A}{B} = \frac{314159265359}{100000000000}$; this large Fraction is reduced to

E

one

one much less, by Division, as in Article 29, for thereby you get the Quotients 3, 7, 5, 1, 292; but this last being so large, shews the four first 3, 7, 5, 1, are sufficient; for of them you form the following Ratios,

$$\frac{3}{1}, \frac{22}{7}, \frac{333}{106}, \frac{355}{113} \text{ (see Articles 31, 32, \&c.)}$$

Now this last Fraction reduced to Decimals, gives 3.1415929 to 1, for the Ratio of A to B; which is true to 7, and almost to 8 Places of Decimals.

47. For a *second* EXAMPLE; The Period of the EARTH about the SUN is 565 D. 5 H. 49'. 25" = 525950' = A; and that of VENUS 224 D. 16 H. 41'. 45" = 323562' = B; or $\frac{A}{B} = \frac{525950}{323562}$; then by Division we get the following Quotients, viz. 1, 1, 1, 1, 2, 30; which (by proceeding as the THEORY directs) will produce the following Ratios, viz. $\frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{13}{8}, \frac{395}{243}$; the two last of which come near the Value of A to B.

48. The Fraction or Ratio $\frac{13}{8} = \frac{52}{32}$, is that which HUGENIUS chose for the Wheels of the EARTH and VENUS in his *Automaton*; but as this is less than the real Value of $\frac{A}{B}$, so it will make Venus too slow in the Orrery by 3°. 36' in 20 Years.

49. If we take the Fraction $\frac{395}{243}$, it will a little exceed the Ratio of A to B, but will be much nearer to it than the former; for it will accelerate the Motion of Venus in the Orrery only 6'. 29". in 20 Years. For in the Heavens the Ratio of B to A is 323562 : 525950 :: 243 : 394, 9965 which is less than that in the Orrery 243 : 395, by only 0,0035 in 243 Years; but the 0,0035

Part

Part of a Revolution is equal to $1^{\circ} 15' 36''$; therefore $243 : 1^{\circ} 15' 36'' :: 20 : 6' 28'' 40'''$, as above. A Degree of Accuracy that must satisfy even beyond Expectation. And in this Manner the WHEEL-WORK is calculated for all the other PRIMARY PLANETS; see Page 4.

50. The *third* EXAMPLE shall be for the MOTION of the MOON, whose mean *synodical* MONTH is 29 D. 12 H. 44' 3" = 42524'; and the Earth's Revolution being

525950', we have the large Ratio, $\frac{525950}{42524} = \frac{21038}{1701}$;

which gives the Quotients 12, 2, 1, 2, 1, 1, 6, 3; from

these we raise this Series of Ratios $\frac{12}{1}, \frac{25}{2}, \frac{37}{3}, \frac{99}{8},$

$\frac{136}{11}, \frac{235}{19}, \frac{1781}{144}$; and this last Ratio will furnish a Set

of Wheels for the MOTION of the MOON to the Exactitude mentioned in Page 7. In the same Manner are found the Wheels for her NODE and APOGEE.

51. Lastly; For a 4th EXAMPLE, JUPITER's Period is 4332 D. 12 H. 20' = 6238820'; and that of his 4th Satellite is 16 D. 16 H. 32' = 24032'; whence the large

Ratio $\frac{6238820}{24032} = \frac{1559705}{6008}$; which gives the Quotients

259, 1, 1, 1, 1, 8, &c. and from them arise these ap-

proximating Ratios, $\frac{259}{1}, \frac{260}{1}, \frac{519}{2}, \frac{779}{3}, \frac{1298}{5},$

$\frac{11163}{43}$; this last will give the Wheels for a *primum Mobile*

for Jupiter's four Moons: And thus you proceed for Saturn's five Moons and Ring.

From

From these Examples the Excellency and Universality of this THEORY is manifest. This is one of the many noble Inventions of the late learned Mr. COTES, elucidated by Dr. SAUNDERSON, and is here, for the first Time, applied to the MECHANISM of the Planetary MOTIONS.

ADVERTISEMENT.

AS the ORRERY here proposed consists of a great Variety of Parts and Machinery, so it may be made in various Degrees of Perfection, according as Gentlemen are disposed to have it; and to prevent Trouble, the Prices of the several Parts, less or more combined, will be as stated below.

First; The PLANETARIUM only (as in Fig. 1.)	12 12 0
Secondly; The PLANETARIUM and TELLURIAN (Fig. 1. and 2.)	21 0 0
Thirdly; The PLANETARIUM, TELLURIAN, and LUNARIUM compleat (as in Fig. 1, 2, 3.)	35 0 0
Fourthly; The PLANETARIUM, TELLURIAN, LUNARIUM, Jovian SYSTEM, and HELIO- TROPE	55 0 0
Fifthly; The ORRERY compleat, with all the Six Parts (mention'd Page 1, and 2)	105 0 0
Sixthly; The same, with all the additional MA- CHINERY (mentioned Page 10 and 11)	157 10 0

12 JY 64

E R R A T A.

- Page 3. Line 17. for *Ptolomaic*, read *Ptolemaic*;
Page 7. in the last Line, read *Heliocentric*.

